Multiple Methods for Solving Equations

There are many ways to think about and solve mathematical problems. In this lesson we will learn a variety of different ways to think about and solve equations, including:

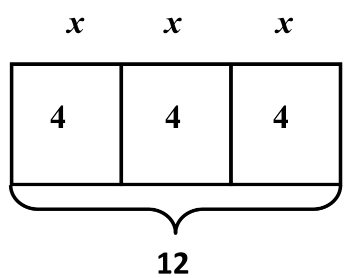
- Inverse Operations
- Bar Models
- Decomposition (and one example of two column proofs)
- Algebra Tile Models

Example 1: Solve: $3x = 12$

Inverse Operations:

$$3x = 12$$
$$\frac{3x}{3} = \frac{12}{3}$$
$$x = \frac{2 \bullet 2 \bullet 3}{3}$$
$$x = 4$$

Bar Model:



$\therefore x = 4$

Decomposition Proof:

STATEMENT	REASON
$3x = 12$	Given
$x + x + x = 4 + 4 + 4$	Definition of multiplication
$x = 4$	Definition of equality

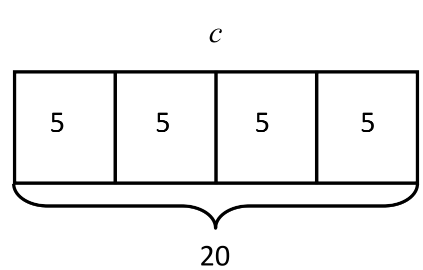
Start to show two column proofs to frontload the concept for use in the derivation of the Quadratic Formula by completing the square and in Geometry.

Example 2: Solve: $\frac{c}{4} = 5$

Inverse Operations:

$$\frac{c}{4} = 5$$
$$4\left(\frac{c}{4}\right) = 4(5)$$
$$c = 20$$

Bar Model:



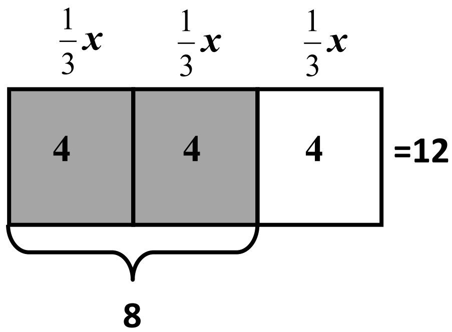
$\therefore c = 20$

Example 3: Solve: $\frac{2}{3}x = 8$

Inverse Operations:

$$\frac{2}{3}x = 8$$
$$\frac{3}{2}\left(\frac{2}{3}x\right) = \frac{3}{2}(8)$$
$$x = 12$$

Bar Model:



$\therefore x = 12$

Decomposition:

$$\frac{2}{3}x = 8$$
$$\frac{1}{3}x + \frac{1}{3}x = 4 + 4$$
$$\frac{1}{3}x = 4$$
$$x = 12$$

Note: You can show if $\frac{1}{3}$ of a number is 4, the number is 12 with a bar model.

You Try: **Solve using at least two different methods.**

1) $4x = 20$

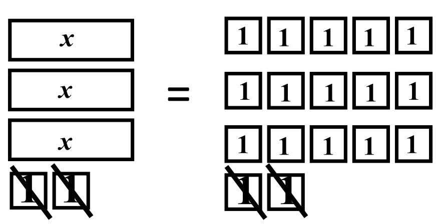
2) $\frac{3}{5}x = 30$

Example 4: Solve: $3x + 2 = 17$

Traditional:

$$3x + 2 = 17$$
$$3x + 2 - 2 = 17 - 2$$
$$3x = 15$$
$$\frac{3x}{3} = \frac{15}{3}$$
$$x = 5$$

Connection Between Algebra Tiles and Decomposition:



$\therefore x = 5$

$$3x + 2 = 17$$
$$3x + 2 = 15 + 2$$
$$3x = 15$$
$$x + x + x = 5 + 5 + 5$$
$$x = 5$$

You Try: **Solve using at least two different methods.**

1) $6x - 3 = 9$

Example 5: Solve: $3(x+1)=15$

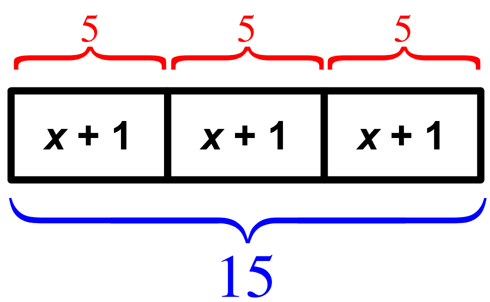
Decomposition (Expansion):

$$3(x+1)=15$$
$$(x+1)+(x+1)+(x+1)=5+5+5$$
$$x+1=5$$
$$x+1=4+1$$
$$x=4$$

Inverse Operations:

$$3(x+1)=15$$
$$\frac{3(x+1)}{3}=\frac{15}{3}$$
$$x+1=5$$
$$x+1-1=5-1$$
$$x=4$$

Bar Model



Distribute then Inverse Operations:

$$3(x+1)=15$$
$$3x+3=15$$
$$3x+3-3=15-3$$
$$3x=12$$
$$\frac{3x}{3}=\frac{12}{3}$$
$$x=4$$

Example 6: Solve: $3(4b-7)=27$

Decomposition (Expansion):

$$(4b-7)+(4b-7)+(4b-7)=9+9+9$$
$$(4b-7)=9$$
$$4b-7=9+7-7$$
$$4b=16$$
$$b+b+b+b=4+4+4+4$$
$$b=4$$

Inverse Operations:

$$\frac{3(4b-7)}{3}=\frac{27}{3}$$
$$(4b-7)=9$$
$$4b-7+7=9+7$$
$$4b=16$$
$$\frac{4b}{4}=\frac{16}{4}$$
$$b=4$$

You Try: Solve using at least two different methods.

1) $2x+1=7$

2) $4(3x+2)=32$

It is important to show multiple approaches to solving equations to:

- Develop relational thinking
- Develop flexible thinking
- Develop student mathematical intuition
- Provide scaffolding to ensure more students are successful